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Optimum Scenarios Evaluation of Reverse Logistics Systems under Influence of Uncertainties: Decisions made on Operating Costs

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***Abstract*— A time-discrete, constrained, Linear Quadratic Gaussian (LQG) production planning problem is formulated to develop a production plan with sub-optimal levels of production and remanufacturing for a single product. With the objective to define a strategy of remanufacturing used product, estimated return rates are used to provide production scenarios based on this plan. Nowadays, specific legislation is applied to many industrial sectors regard to the return of used products. Thus, motivated by environmental factors and a shortage of raw materials, partial or total reuse of return products are a high priority on business's agenda of many companies. This paper uses an approach of literature to solve a production-planning problem of a dynamic system that includes a reverse channel, with a remanufacturing facility. It is assumed that fluctuations of demand for serviceable products are approximated by stationary normal random variables. Thus, the constrained LQG problem here considered is converted to an equivalent but deterministic that can be solved by any quadratic programming method from literature, providing, as a result, sub-optimal inventory and production plans. Managers can use such plans as scenarios to evaluate future opportunities as costs' reduction, for instance. A simple *usecase* illustrates such ideas.**

I. Introduction

In the literature, we found different definitions for reverse logistics. Generally speaking, however, processes related to actions of recycling, remanufacturing, solid waste disposal, and handling properly hazardous materials in order to protect human being and the environment define reverse logistics. These processes require typical activities of planning and controlling of used material flow through the reverse channel of any responsible supply chain. The main goal of reverse logistics is to move the used products from their local use to a safe destination, having in mind the possibility of adding partial or total value to them. As a result, of this aggregation value, we can understand the reverse logistics as being a recycling or remanufacturing process of used products.

Thus, for today and next future, reverse logistics represents an important part of the supply chain process. There are political and economic reasons to recover used products. For example, international severe laws related to destiny of unserviceable products; high costs to collect used products from customers; enormous costs to storage or disposal used products; and, the fact of returnable products can be converted to reusable product, or some parts can be extracted to repair products used by companies. Thus, companies have shown great concern about the destiny of used-products.

Issues and problems related to reverse logistics are largely found in journals and books of literature. Most of them are based on quantitative models that are used to represent remanufacturing and recycling activities in the reverse channel. We consider here a classification based on types of problem provided by Fleischmann et al. [1] that are directly related to quantitative models for reverse logistics. In their typology, authors focus on three types of problems: (*i*) reverse distribution problems; (*ii*) inventory control problems in systems with return flows of products; and (*iii*) production planning problem with reuse of parts and materials. In short, the first type is associated to collecting and transporting of used products and packages. According to above mentioned authors, "the reverse distribution can take place through the original forward channel, through a separate reverse channel, or through combinations of the forward and the reverse channel." The second one is related to production planning and control mechanisms that allow repairing used products and bring them back to the marketplace. At last, the third type considers production-planning problems of reusing parts and products without remanufacturing. It is worth mentioning that there are different optimization approaches to deal with these types of problems, see [3] and [6].

In this paper, our interest is to discuss how to provide production plans by solving a stochastic optimization problem subject to dynamic systems with return flows and physical probabilistic constraints. According to Fleischmann topology [1], our paper focuses on inventory and production planning category. In fact, the paper proposes the use of an approach available in the literature, see [5], but that is here applied to a problem of planning of a supply chain with reverse channel, which includes a remanufacturing facility. Typical examples of product recovery are tires, toner cartridges, TV sets and distribution equipment like bottles, pallets, containers, etc. The paper deals with a typical issue, when sometimes used-products can be recovered for reuse.

Linear Quadratic Gaussian (LQG) problem with constraints on decision variables can be very useful to deal with inventory- production planning problems (see Holt et al [2]). The quadratic criterion can penalize both excesses as well shortage of inventory, for instance. Another important impact of a LQG formulation is the possibility of using constraints explicitly, that is, without penalizing approaches as have been used by other techniques. In our paper, the main objective is to satisfy the demand over the periods of the planning horizon, with minimal costs of holding inventories, manufacturing new products, and remanufacturing or disposal used-products. We focus on showing that by reusing used-products after remanufacturing, companies can obtain competitive advantage due to cost reduction. In the literature, it is possible to find numerous contributions related with this kind of model, the majority of them consider the formulation in a time-continuous pattern; see, for instance, [3] and [4].

Sub-optimal inventory-production plans can be provided by the approach here considered. These plans can be used to provide different scenarios of production, created due to the dynamic and uncertain nature of the system. Indeed, some parameters of the model can change to provide such scenarios. Typical parameters that can be changed to provide these scenarios are the return rate of used products, the delay of return, or both, etc. In this paper, the above parameters will be maintained as constants. Scenarios will be created as a result of the estimated fluctuation of demand for serviceable products. Three situations will be explored: one considering the expected value of demand exactly, while the other two, will take into account maximum and minimum estimated errors of demand forecasting.

The sections of the paper are organized as follows: Section 2 formulates the optimization model to represent the reverse logistics problem. Section 3 discusses a solution for this problem and introduces a simulation scheme for scenarios analysis. In sequence, Section 4 introduces a simple *use case* that illustrates the economic feasibility of finding an optimal balance for a recovery system by mean of a structured procedure that considers optimal scenarios.

II. The Stochastic Production Problem

Two inventory balance systems are illustrated in Figure 1 through their Distribution Centers (DC): the first one is on the forward channel of the supply chain and represents a warehouse of serviceable products (i.e., manufactured and remanufactured products), being denoted here by DC1.

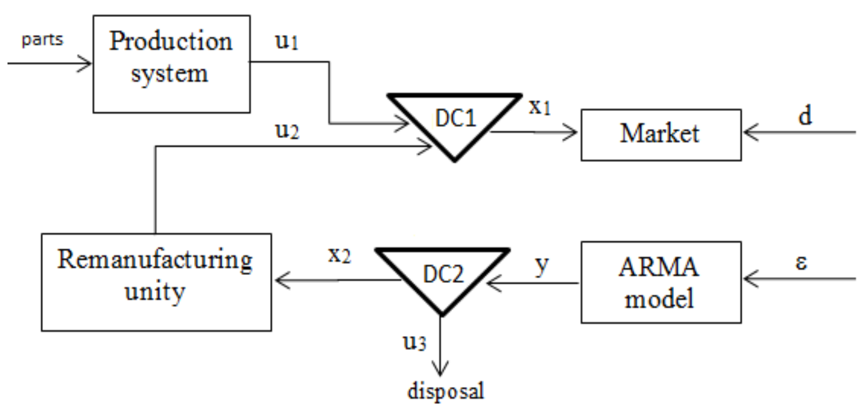

Figure 1.Dynamic system of reverse logistics

The second is on the reverse channel and it represents a warehouse of returnable products, denoted by DC2. Note that the serviceable products must fulfill customer demand for products, which is a merge of new and remanufactured products.

Note that the integrated forward and reverse channels of the supply chain illustrated in the figure 1 can be described as a discrete-time stochastic dynamic system. In fact, it encompasses two state variables x_1 and x_2 (i.e. inventory levels) associated respectively to the serviceable unit (DC1) and the remanufacturing unit (DC2); and three control variables that are related to manufacturing (u_1), remanufacturing (u_2), and discard (u_3) rates. The stochastic nature of the system is due to demand d that is not precisely known, and consequently, has impacted the state variable x_1 that becomes a random variable, as will be discussed later.

Some points to be considered from figure 1 are:

– For each period k, a return rate y is estimated from an Autoregressive, Moving-Average model, denoted as ARMA(p,q), where p is the order of the autoregressive part and q is the order of the moving average part. A time-series of returnable products is used to identify the model.

- Serviceable storage unit (DC1) has lower and upper bound capacity denoted by $\underline{x}_1$ and $\bar{x}_1$, respectively. Note that the parameter $\underline{x}_1$ denotes the safety-stock level defined by the company. Furthermore, returnable storage unit (DC2) has upper bound capacity denoted by $\bar{x}_2$. Usually, returnable level of safety stock is not considered that means that $\underline{x}_2 = 0$.
- Manufacturing and remanufacturing rates (i.e., u_1 and u_2) have upper limits of operational capacity denoted by $\bar{u}_1$ *and* $\bar{u}_2$, respectively. Sometimes, however, planning strategy can impose lower boundary capacities for u_1 and u_2. Thus, it is reasonable to include $\underline{u}_1$ and $\underline{u}_2$ into the model to represent these lower levels, respectively.
- Disposed rate u_3 represents returnable products that are spoiled and, therefore, cannot be remanufactured. There are two main reasons to discard used-products: the first has a *technical* justification, which is related to inappropriate returned products for remanufacturing activities; and the second has a *financial* justification, in which remanufacturing all products can significantly raise the inventories, and as a result increase production costs.

The stochastic control system is described by the following two difference equations, which represent the inventory balance systems related to forward and reverse channels of the supply chain:

$$x_1(k+1) = x_1(k)+u_1(k)+u_2(k)-d(k) \quad (1)$$

$$x_2(k+1) = x_2(k)-u_2(k)-u_3(k)+y(k) \quad (2)$$

The demand $d(k)$ is a Gaussian variable with mean and variance given by $\hat{d}(k)$ and σ_d^2. As a result of random nature of d, the inventory system (1) is a stochastic process, then the state $x_1(k)$ is also a random variable with mean and variance given respectively by $\hat{x}_1(k)$ and $\sigma_{x_1}^2(k) = k.\sigma_d^2$.

The return level $y(k)$ is provided from an Autoregressive ARMA(p,q) process:

$$(1-\phi_1 z^{-1} - \ldots - \phi_p z^{-p})y(k) = (1-\varphi_1 z^{-1} - \ldots - \varphi_q z^{-q})\varepsilon(k) \quad (3)$$

where sequences $\{\phi_1,\phi_2,\ldots,\phi_p\}$ and $\{\varphi_1,\varphi_2,\ldots,\varphi_q\}$ denote the parameters of AR and MA processes, respectively. The p and q are orders of these respective processes. The input of the ARMA model (see figure 1) is given by the variable $\varepsilon(k)$ that denotes a white noise originated from a normal process.

The following forecast model for t-periods ahead gives the estimation of returnable products:

$$\begin{aligned}\tilde{y}_t(k) = y(k+t) = \phi_1 y(k+t-1) + \ldots + \phi_p y(k+t-p) + \\ \varepsilon(k) - \varphi_1\varepsilon(k+t-1) + \ldots + \varphi_p\varepsilon(k+t-q)\end{aligned} \quad (4)$$

Finally, an optimal sequential production-inventory policy $\{u_1(k)$, $u_2(k)$ and $u_3(k)$ with $k = 0,1,2,\ldots,T$-1$\}$ can be provided by solving a Linear Quadratic Gaussian (LQG) problem with constraints on decision variable. This problem is considered to represent a reverse logistics problem whose system is given by Figure 1. It is formulated as follows:

$$\begin{aligned}
&\underset{u_1,u_2,u_3}{Min}\ h_1 E\left\{x_1^2(T)\right\} + h_2 x_2^2(T) + \{\sum_{k=0}^{T-1} h_1 E\left\{x_1^2(k)\right\} + h_2 x_2^2(k) + \\
&\quad c_1 u_1^2(k) + c_2 u_2^2(k) + c_3 u_3^2(k)\}\Big\} \\
&s.t. \\
&\quad x_1(k+1) = x_1(k) + u_1(k) + u_2(k) - d(k) \\
&\quad x_2(k+1) = x_2(k) - u_2(k) - u_3(k) + y(k) \\
&\quad Prob.\left[\underline{x}_1 \le x_1(k) \le \bar{x}_1\right] \ge 1-\alpha \\
&\quad \underline{x}_2 \le x_2(k) \le \bar{x}_2 \\
&\quad \underline{u}_1 \le u_1(k) \le \bar{u}_1
\end{aligned} \quad (5)$$

where h_1 and h_2 denote the holding costs of DC1 and DC2. The coefficient c_1, c_2 and c_3 denote costs for manufacturing, remanufacturing and disposal. The symbols $E\{.\}$ and $Prob.[.]$ denote the expectation and probability operators respectively. The index α is a probabilistic index and represents the customer satisfaction level; see [5]. Note that DC1 and DC2have upper and lower boundaries of stock. The safety-stock in DC1 depends on the customer satisfaction α provided by the manager. Note also that $\bar{u}_1$ and $\bar{u}_2$ are the upper levels of production capacities.

The linearity of systems (1-2), and the convexity of criterion of problem (5) allow us to use the classical *certainty equivalence* principle to replace the stochastic problem (5) by an equivalent deterministic problem [6]. In fact, assuming the serviceable inventory system (1) approximated by Gaussian process [7], where x_1(k) is a Gaussian variable with first and second statistics moments $\hat{x}_1(k)$ and $k.\sigma_d^2$ [8], a constrained Linear Quadratic (LQ) problem, denoted as Mean Value Problem (MVP), can be formulated as follows:

$$\begin{aligned}
&\underset{u_1,u_2,u_3}{Min}\ h_1\hat{x}_1^2(T)+h_2x_2^2(T)+\left\{\sum_{k=0}^{T-1}h_1\hat{x}_1^2(k)+h_2x_2^2(k)+\right.\\
&\qquad \left. c_1u_1^2(k)+c_2u_2^2(k)+c_3u_3^2(k)\}\right\}+\cdot h_1\cdot\left(\sum_{k=0}^{T}k\right)\cdot\sigma_D^2\\
&s.t.\\
&\qquad \hat{x}_1(k+1)=\hat{x}_1(k)+u_1(k)+u_2(k)-\hat{d}(k) \qquad (6)\\
&\qquad x_2(k+1)=x_2(k)-u_2(k)-u_3(k)+\tilde{y}_t(k)\\
&\qquad \underline{x}_1+\sqrt{k}\cdot\sigma_d\cdot\Phi_x^{-1}(\alpha)\le\hat{x}_1(k)\le\bar{x}_1-\sqrt{k}\cdot\sigma_d\cdot\Phi_x^{-1}(\alpha)\\
&\qquad \underline{x}_2\le x_2(k)\le\bar{x}_2\\
&\qquad \underline{u}_1\le u_1(k)\le\bar{u}_1\\
&\qquad \underline{u}_2\le u_2(k)\le\bar{u}_2
\end{aligned}$$

where Φ_x^{-1} denotes the inverse distribution of probability; $\hat{d}(k)=E\{d(k)\}$ is the mean value of demand; $\tilde{y}_t(k)$ is given by (4); and $E\{x_1^2(k)\}=\hat{x}_1^2(k)+k\cdot\sigma_d^2$, $\sigma_x(k)=\sqrt{k}\cdot\sigma_d$ [8].

At last, it is worth mentioning some advantages of using quadratic cost: a) it penalizes equally, both positive (i.e., excess) and negative (i.e., backlogging), variations of decision variables; and b) it induces high penalties for large deviations of the decision variables from the origin but relatively small penalties for small deviation, see [6].

The equivalent deterministic problem (6) is easier to be solved than the original one (5) that is very time-consuming particularly when stochastic dynamic programming is applied, see [6]. This is the main advantage of using such an approach. In fact, it provides quasi-optimal (i.e. sub-optimal) solutions. Bertesekas [6] has shown the applicability of this approach for practical problems. In next section, we discuss the use of this approach to deal with production scenarios that help managers to make their decisions more precisely.

III. OPTIMAL SCENARIOS

Figure 2 exhibits a schematic diagram that represents the application of optimal plan provided by the policy for running systems (1)-(2). From this diagram, it is possible to visualize the main steps of simulation scheme. From this simulation process, it is possible to provide production scenarios related to company resources. Indeed, varying some parameters of the problem (5) and running the simulation process (see Figure 2), production scenarios are created in order to help managers to make decisions.

Note that a one-step ahead forecast for rate of returnable products (i.e. $\tilde{y}_1(k)$) and prices of the criterion given in (6) are considered here to generate production scenarios, which allow users evaluating and making proper decisions. Let's consider some comments:

(a) Forecasting of returnable products: from a historical of returnable products (time-series), an ARMA(p,q) model (3)can be identified. Then a forecasting model of one-steps ahead $\tilde{y}_1(k)$ is developed from (4) in order to provide estimations about future levels of returnable products. Associated to this estimation, an forecasting interval is considered to determine upper and lower limits for future estimated values of $y(k)$, which is given by $\tilde{y}_1(k)$, see (4). A parameter $\delta \in [0,1]$ should be chosen by manager; it allows calculating the upper and lower limits of forecasting. As a result, three scenarios can be proposed:

- first scenario follows the estimated values of returnable products, that is $\{ \tilde{y}_1(k)$, k=0,1,…,T-1};

- second scenario considers the minimum estimated values given by the sequence $\{ \tilde{y}_{min}(k) = \tilde{y}_1(k)\cdot(1-\delta)$, k=0,1,…,T-1};

and at last

- third scenario considers the maximum values given by $\{ \tilde{y}_{max}(k) = \tilde{y}_1(k)\cdot(1+\delta)$, k=0,1,…,T-1}.

(b) *Prices* (h_i and c_i): they allow evaluating price policies related to inventory and production of manufactured and remanufactured products; thereby enabling the decision-maker to identify opportunities for business in the medium and long-term.

(c) Other important points are:

- The fluctuation of demand over periods of planning horizon is provided from a random generator, which uses first and second statistical moments taken from the history of demand; and

- The Mean Value Problem module provides optimal update policies (i.e., u_1, u_2 and u_3) that are inputs to serviceable (DC1) and returnable (DC2) units. Note that problem (6) can be solved in rolling horizon scheme. Thus for each new period as soon as new observations of inventory levels (i.e. $x_1(k)$ and $x_2(k)$) are measured, the problem (5) is solved and, as a result, productions and disposal rates for that period (i.e. $u_1(k)$, $u_2(k)$ and $u_3(k)$} are provided. This policy is immediately applied to systems (1)-(2), see Figure 2.

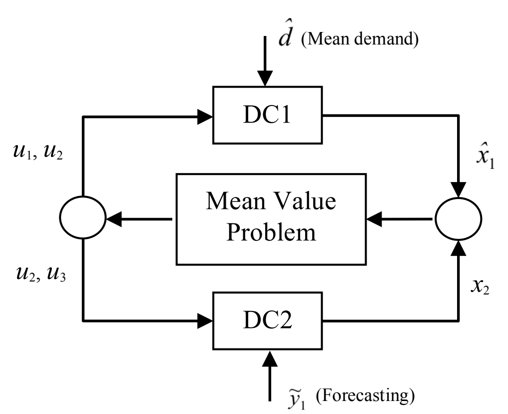

Figure 2. Simulation scheme using Mean Value Problem (6)

IV. USE CASE

Let's consider now a hypothetical situation of a simple company that manufactures a given product that is returned to company after being used. It is assumed that the product is collected, checked and then decided if it should be remanufactured or disposed. The company runs in a make-to-stock production, having inventories units in forward and reverse channels of the supply chain, as showed in Figure 1.

It is assumed that the demand for this product is stationary, which means that its fluctuation can be estimated with good accuracy over periods of the planning horizon. The cost of remanufacturing returnable products that involves disassembling, overhaul and replacement is supposed lower than the cost of manufacturing news.

In this *use case*, the constrained LQ problem (6) is considered with an objective of developing a production-inventory plan over a planning horizon of T=4 (four months). Production scenarios are provided from (6) by considering a sequence of estimated demand and its error deviation. The idea is to help the manager to analyze the reverse logistics policy of the company. The problem's data are listed below.

TABLE 1 AVERAGE AND STANDARD DEVIATION OF DEMAND

Periods	Jan (1)	Feb (2)	Mar (3)	Apr (4)
Mean	42	45	43	48
Deviation	$\sigma_d \approx 3.0$			

Other data:

- $t = 1$ month (forecasting horizon)
- $\delta = 20\%$ (forecasting error)
- $x_1(0) = 20$ initial inventory of manufacturing
- $x_2(0) = 10$ initial inventory of remanufacturing
- $\underline{x}_1 = 0$ (there is no safety-stock on hand)
- $h_1=2$; $h_2=1$ (inventory costs)
- $c_1=3$; $c_2=2$; and $c_3=1$ (productions and disposal costs)

The customer satisfaction performance index is assumed as $\alpha = 0.84$, which means that orders of customers for serviceable product is attempted 84% of time. This percentage of success is mathematically guaranteed by the establishment of a safety stock in the storage unit of serviceable products (DC1), which is calculated from the probabilistic constraint given in (6). This safety-stock is given by:

$$\underline{x}_1(k) \geq \sqrt{k} \cdot \sigma_d \cdot \Phi^{-1}(\alpha) = 3 \cdot \sqrt{k} \tag{7}$$

where $\sigma_d = 3$ and $\Phi^{-1}(0{,}84) = 1$.

Evaluation of three scenarios: the estimation of returnable products is given one-step ahead forecasting models derived from an autoregressive model (AR(2)). The sequence of forecasting values of returnable products and correspondent sequences with upper and lower values based on forecast error δ=0.20 are exhibited in Table 2. From these forecasting sequences, three scenarios are analyzed.

TABLE 2 FORECASTING PRODUCT RETURNS

Periods	Jan (1)	Feb (2)	Mar (3)	Apr (4)
$\tilde{y}_{min}$	29	32	30	34
$\tilde{y}_1$	37	40	38	43
$\tilde{y}_{max}$	44	48	45	51

Scenario (1): In this scenario, a one-step ahead forecast sequence $\tilde{y}_1$, given in the Table 2, is considered in the solution of the problem (6). As a result, it is observed that the mean demand $\hat{d}$ is almost entirely met by the reverse chain; see Figure 4 that shows the rate of remanufacturing.

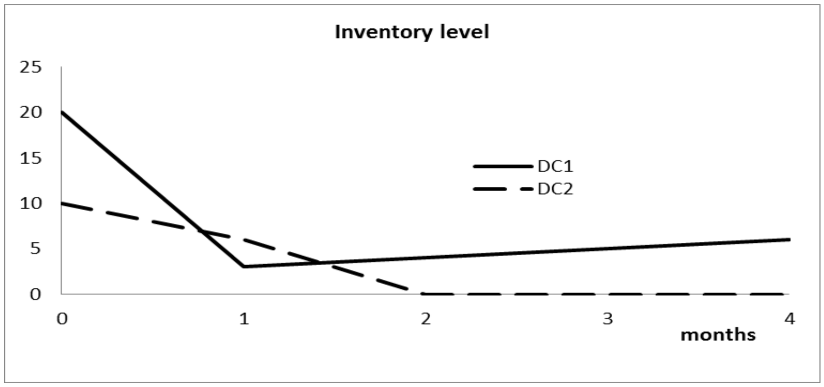

Figure 3. Optimal inventoy levels with $\widetilde{y}_1$

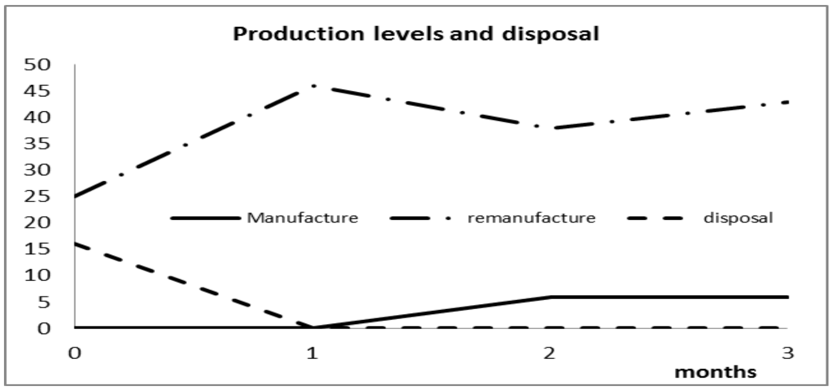

Figure 4. Optimal control policies with $\widetilde{y}_1$

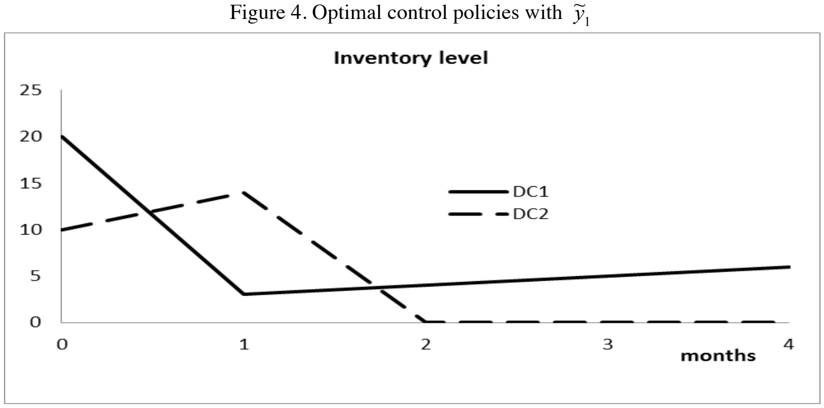

Figure 5. Optimal inventory levels with $\widetilde{y}_{min}$

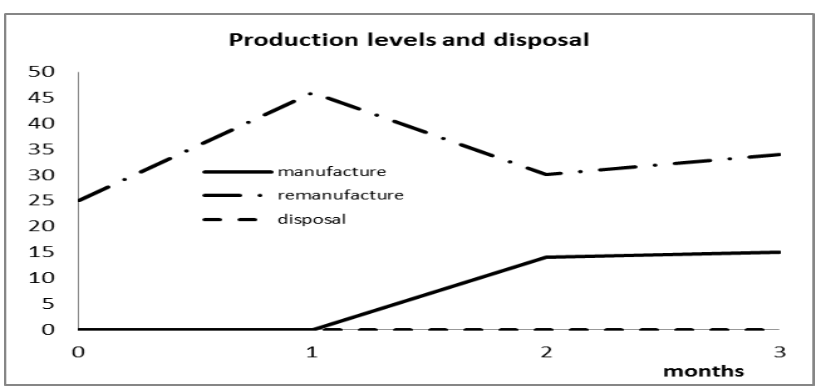

Figure 6. Optimal production policies with $\widetilde{y}_{min}$

This fact becomes clear when comparing the product level remanufactured with the level of new products. In Figure 4, it can be observed that in the period of four months, there were 84% of remanufactured products, 7% of new manufactured products, and 8% of products placed for disposal. From Figure 3, it is possible to point out that the inventory of used products in the unit DC2 was entirely consumed, while the serviceable inventory of unit DC1 preserves a small growing of safety-stock to accommodate future uncertainties that is, ensuring the customer satisfaction close to 84%.

Scenario (2): this scenario that is illustrated by Figures 5 and 6 considers a sequence of values that represents 80% of forecasted values $\tilde{y}_1$ (i.e. $\tilde{y}_{min}$ sequence available in Table 2).

Figure 5 shows that the supply policy for DC1 is also based on remanufactured products, but let us point out that from the second period (see Figure 6), new manufactured products are required to meet the demand. In fact, taking in percentage terms, it can be observed that 82% of serviceable products are remanufactured (i.e., u_2) and, while only, 18% are effectively manufactured as new products (i.e., u_1). Note that some serviceable products are left as safety-stock in DC1 to guarantee the customer satisfaction performance related to prompt delivery of products.

Scenario (3): In this scenario is considered that the estimates of return rate of used products are set equal to upper limits $\tilde{y}_{max}$, which is 20% higher than the forecast values in the sequence $\tilde{y}_1$ found in the Table 2.Note from Figure 8 that all production is oriented to meet the mean demand and it is completely associated with the remanufacturing process. In fact, as shown in Figure 7, the demand quickly consumes the initial inventory levels of DC1 and DC2 units. However, the DC1 unit has a small, but growing safety stock given by (7) to avoid surprises, such as an unexpected order of customers. It is important to see that new products are not manufactured in this scenario (i.e., $u_1(k) = 0, \forall k$).

The costs related with the three scenarios are given bellow:

TABLE 3. COSTS VERSUS SCENARIOS ($)

Costs ($)	**Scenario 1**	**Scenario 2**	**Scenario3**
DC1	$ 216	$ 216	$ 216
DC2	$ 6	$ 14	-
Manufactured cost	$ 36	$ 87	-
Remanufactured cost	$ 304	$ 270	$ 328
Disposal cost	$ 16	-	$ 34
Total cost	**$ 578**	**$ 587**	**$ 578**

If we eliminate the reverse channel of the system shown by Figure 1, the total cost will be equal to $ 708. Comparing this cost with those provided in Table 3, we conclude that a reverse policy is a profitable activity for the company. However, advantages for using reverse logistics channel will depend on the associated costs. In fact, the costs for collection and handling of used products are assumed negligible in the evaluated scenarios. Looking at Table 3, we can conclude that increasing by 20% in the expected rate of used-products return did not produce any reduction on the total costs of the company, which remained at $ 578 (see scenarios 1 and 3). Increasing the return rate by 50% also did not bring any benefit to the company that had a total cost of $ 626. Thus, we can deduct that increasing the return collection rate may have an adverse effect on reducing the company's costs.

V. CONCLUSION

A linear quadratic Gaussian (LQG) model with constraints was formulated to provide optimal plans for manufacturing, remanufacturing and disposal variables. Through the analysis of three scenarios generated by these plans, it was possible to compare some situations related to the use of a reverse channel with remanufacturing facility. We believe that the use of optimization and optimal control techniques can help manager to make strategic decisions related to the process of planning a reverse channel to company.

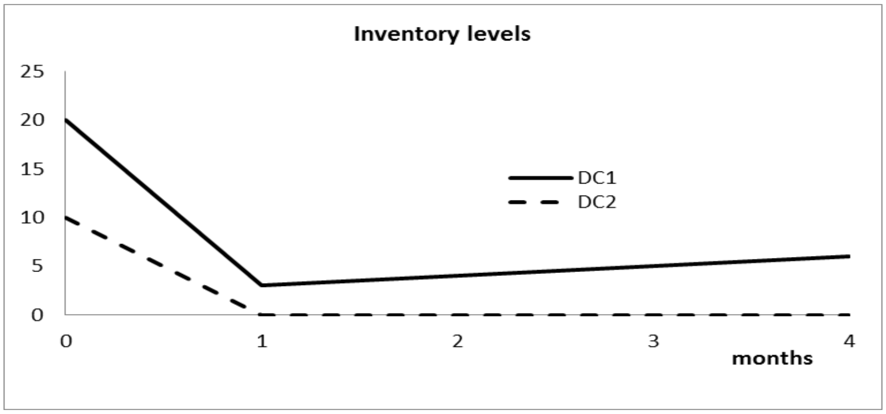

Figure 7.Optimal inventory levels with $\tilde{y}_{max}$

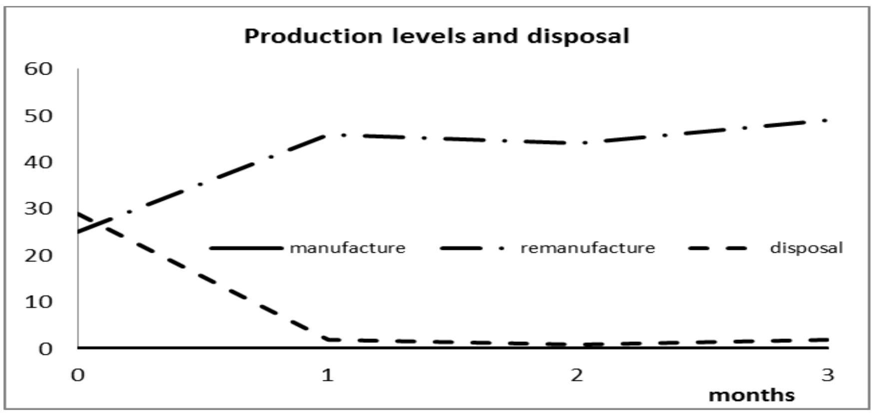

Figure 8. Optimal production policies with $\tilde{y}_{max}$

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